Quantum generation of non-trivial spatial topology in post-inflationary universes

Alexei A. Starobinsky

Landau Institute for Theoretical Physics RAS,
Moscow-Chernogolovka, Russia
and Joint Institute for Nuclear Research, Dubna,
Russia

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Spatial topology of a FRW universe

From observations, at large scales $R \gtrsim 10$ Mpc:

$$ds^2 = dt^2 - a^2(t)d\Sigma^2$$

up to small corrections $\sim 10^{-5}$, where $d\Sigma^2$ describes the constant curvature 3-space.

I. $k = 0$: flat 3-space.
Favoured by observations: most recent limits using CMB WMAP7 and SPT data $\Omega_k = -0.003^{+0.014}_{-0.018}$ (K. T. Story et al., arXiv:1210.7231).
Possible non-trivial topological types:
a) $T^1, \quad x \leftrightarrow x + L_1$
b) $T^2, \quad x \leftrightarrow x + L_1, y \leftrightarrow y + L_2$
c) $T^3, \quad x \leftrightarrow x + L_1, y \leftrightarrow y + L_2, z \leftrightarrow z + L_3$
18 types as a whole (including the Möbius strip, etc.).

II. $k = 1$: positive spatial curvature.

a) Elliptic topology

b) S^3/Z_N

c) Dodecahedron topology

Countable number of types.

III. $k = -1$: negative spatial curvature.

a) Non-compact,

e.g. the horn topology

$d\Sigma^2 = dx^2 + e^{2x}(dy^2 + dz^2), \quad y \leftrightarrow y + L_1, z \leftrightarrow z + L_2$

b) Compact

$D_{max} \gtrsim R$

Uncountable number of types.

Observational search for a non-trivial spatial topology

The most sensitive method: using CMB fluctuations.
1. The matched circles method (N. Cornish et al., 1998).
Circles: lines of the intersection of the last scattering surface with boundaries of elementary domains.
2. The statistical method: uses specific features in CMB temperature anisotropy multipoles a_{lm}.
E.g., for T^3 with $L_1 = L_2 = L_3 \ll R_{LSS}$, multipoles with $l < 4\frac{R_{LSS}}{L_1}$ are suppressed.
First search (I. Yu. Sokolov, 1993; A. A. Starobinsky, 1993; D. Stevens, D. Scott and J. Silk, 1993): $L_1 \gtrsim 0.75 R_{LSS}$.
The most recent lower limit (P. Bielewicz and A. J. Banday, 2011): $L_1 \gtrsim 2R_{LSS} \approx 28$ Gpc.

More subtle features for T^1 and T^2, or for T^3 with strongly different topological scales (A. A. Starobinsky, 1993).

1. T^2, or T^3 with $L_1 \sim L_2 \ll R_{LSS} \lesssim L_3$:
the previous answer + additional axisymmetric component $\frac{\Delta T}{T}(\theta)$.

2. T^1, or T^3 with $L_1 \ll R_{LSS} \lesssim L_2, L_3$:
the answer for the cubic T^3 + an additional component with the planar (mirror) symmetry $\frac{\Delta T}{T}(\theta, \phi) = \frac{\Delta T}{T}(\pi - \theta, \phi)$.

First search (A. de Oliveira-Costa, G. F. Smoot, A. A. Starobinsky, 1996): $L_{min} \gtrsim 0.5 R_{LSS}$. The present lower limit is about $L_{min} \gtrsim 1.2 R_{LSS}$ (no full agreement between different authors).

Search for hidden features in CMB fluctuations

Search in the similar direction:
F. Finelli, A. Gruppuso, F. Paci, A. A. Starobinsky, Searching for hidden mirror symmetries in CMB fluctuations from WMAP 7 year maps, JCAP 1207, 049 (2012).

Two different axes are found for which the CMB intensity pattern is anomalously symmetric (or anti-symmetric) with respect to reflection in planes orthogonal to them at the 99.84 (99.96)% confidence level if compared to a result for an arbitrary axis in simulations without the symmetry. The directions of these axes are close to that of the CMB kinematic dipole and nearly orthogonal to the ecliptic plane, respectively. These axes are not the axes of axial symmetry.

If instead the real data are compared to those in simulations taken with respect to planes for which the maximal symmetry is generated by chance, the confidence level decreases to $92.39(76.65)\%$. By when the effect in question translates into the anomalous alignment between normals to planes of maximal mirror (anti)-symmetry and these natural axes.

The symmetry anomaly is almost entirely due to low multipoles $l \leq 10$, so it may have a cosmological and even primordial origin. Contrary, the anti-symmetry anomaly is mainly due to intermediate multipoles $10 < l < 35$ that, combined with its correlation with the ecliptic plane, probably suggests its non-fundamental nature.

Theoretical predictions

1. In classical cosmology, spatial topology is given as an initial condition at the cosmological singularity and does not change in the cause of non-singular evolution in future - the Geroch theorem.
2. Initial topology can be produced by quantum vacuum tunneling, e.g. using some instanton solution having a non-trivial spatial topology.
The instanton describing creation of a universe with the spatial T^3 topology "from nothing", was first constructed in Ya. B Zeldovich and A. A. Starobinsky, 1984.

However, all this changes radically in the inflationary scenario of the early Universe when backreaction of quantum-gravitational fluctuations of an inflaton field on the evolution of a background space-time metric is taken into account.

Spatial topology of post-inflationary universes

Backreaction of quantum-gravitational inhomogeneous fluctuations of an inflaton field driving inflation results in the total number of local e-folds during inflation dependent on $\mathbf{r}$. As a result, after inflation the space-time metric at sufficiently large scales has the form

$$ds^2 = dt^2 - a^2(t)e^{2\zeta(\mathbf{r})}d\mathbf{r}^2 + \text{small terms}$$

Now, the quantity $e^{2\zeta(\mathbf{r})}$ may become infinite at some 2-D hypersurfaces. Then the approximate equal energy density hypersurface $t = const$ is no more a Cauchy hypersurface of the whole space-time. Thus, the Geroch theorem is no more applicable to it, and it may dynamically acquire a non-trivial spatial topology.
In fact, the function $e^{2\zeta(\mathbf{r})}$ even becomes fractal near 2-D surfaces where it diverges. Thus, the resulting spatial topology of $t = const$ hypersurfaces may become fractal, too.

Some simple examples

Let V is the 3-space of the post-inflationary Universe in which we are located and S is its 2-D boundary where $\zeta(\mathbf{r})$ diverges.

1. S is homeomorphic to 2-sphere. Then V has trivial spatial topology.

2. S is homeomorphic to two non-intersecting 2-spheres, one inside the other. Then the spatial topology of V is $R \times S^2$.

3. S is a torus. Then the spatial topology of V is $R^2 \times S$, i.e. T^1.

Conclusions

- Spatial hypersurfaces of approximate large-scale homogeneity of the Universe may have a non-trivial spatial topology.
- In classical cosmology, spatial topology is given as an initial condition at the cosmological singularity and does not change in the cause of a non-singular evolution.
- Observational data (mainly from CMB temperature anisotropies) show that characteristic topological scales exceed the radius of the last scattering surface: no non-trivial spatial topology has been found.

- Inflationary scenario of the early Universe explains this fact. However, it predicts that at very large scales much exceeding the present Hubble radius, spatial topology of 3-D hypersurfaces of approximate homogeneity is generically non-trivial, very complicated and even fractal.
- Classification of all possible types of spatial topology which hypersurfaces of a constant total energy density (less than that at the end of inflation) can have remains an unsolved problem.